Electronic Journal of University of Aden for Basic and Applied Sciences
EJUA-BA Vol. 4 No. 1 (2023)
https://doi.org/10.47372/ejua-ba.2023.1.229
ISSN: 2708-0684

RESEARCH ARTICLE

On Fuzzy $Pre - \Lambda - Sets$ and Fuzzy $Semi - \Lambda - \ Sets$

Radhwan Mohammed Aqeel[1] and Anhar Ahmed Nasser[1,*]

[1] *Dept. of Mathematics, Faculty of Science, University of Aden, Aden, Yemen; E-mail: raqeel1976@yahoo.com*

***Corresponding author: Anhar Ahmed Nasser; E-mail: anharnasser724@gmail.com**

Received: 16 March 2023 / Accepted: 24 March 2023 / Published online: 31 March 2023

Abstract

This paper is devoted to study the notions of fuzzy $pre - \Lambda - set$, fuzzy $semi - \Lambda - \ set$, fuzzy $pre - \ V \ - set$ and fuzzy $semi - \ V \ - \ set$ in fuzzy topological space. We study some properties and discuss the relationships between fuzzy $pre - \Lambda - set$, fuzzy $semi - \Lambda - \ set$, fuzzy $pre - \ V \ - set$ and fuzzy $semi - \ V \ - \ set$ and relevant concepts in fuzzy topological spaces and to investigate some basic yet essential properties.

Keywords: Fuzzy $\Lambda - set$, Fuzzy $pre - \Lambda - set$ and Fuzzy $semi - \Lambda - \ set$, Fuzzy $pre - \ V \ - set$ and Fuzzy $semi - \ V - set$.

1. Introduction

The concepts of fuzzy sets and fuzzy topology were firstly given by Zadeh in [1] and Chang in [2], and after then there have been many developments on defining uncertain situations and relations in more realistic way. The fuzzy topology theory has rapidly began to play an important role in many different scientific areas such as economics, quantum physics and geographic information system (GIS). For instance, Wenzheng Shi and Kimfung Liu mentioned that the fuzzy topology theory can potentially provide a more realistic description of uncertain spatial objects and uncertain relations in [3] where they developed the computational fuzzy topology which is based on the interior and the closure operator. In the fuzzy topology, the weaker forms of fuzzy open sets, which were constructed by the compositions of different combinations of the closure and interior operator, have been studied by several mathematicians [4–8].

The concept of fuzzy $\Lambda \ - \ sets$ and fuzzy $V - \ sets$ ware introduced by M. E. EI-Shafei and A. Zakari, The Arabian Journal of Science and Engineering [9], Halder [10] introduced the concept of fuzzy $R - \Lambda \ sets$ and fuzzy $R - \ V \ sets$ in fuzzy topological space. In (2008), In this paper, for these sets, we will introduce the notions of fuzzy $pre - \Lambda - set$, fuzzy $semi - \Lambda - \ set$, fuzzy $pre - \ V \ - set$ and fuzzy $semi - \ V \ - \ set$ in fuzzy topological space.

2. Preliminaries

Throughout this work, by $(X, \ \tau)$ we mean a fuzzy topological space due to Chang [2] in 1968.The complement of a fuzzy set μ is denoted by μ^c . We start with recalling some lemmas and definitions which are necessary for this study in the sequel.

Definition 2.1. Let $(X,)$ be a fuzzy topological space, then the fuzzy set μ is called:

(i) Fuzzy regular open [11] $\mu = int(cl(\mu))$
(ii) Fuzzy semi open [5,12] (briefly fuzzy s−open) if $\mu \leq cl(int(\mu))$;
(iii) Fuzzy preopen [13] (briefly fuzzy p−open) if $\mu \leq int(cl(\mu))$

The complement of a fuzzy regular open (resp. $s$emi−open and pre−open) set is called fuzzy regular closed (resp. $semi$−closed and pre−closed) set.

Definition 2.2. Let $(X,)$ be a fuzzy topological space, then the fuzzy set μ is called fuzzy $\Lambda{-}set$ [9] if $\mu = \mu^{\Lambda}$, where $\mu^{\Lambda} = \inf \{\upsilon : \ \mu \leq \upsilon, \upsilon \in \tau\}$
The complement of a fuzzy $\Lambda{-}set$ is called fuzzy $V - \ set$.

Lemma 2.3. [9, 14]. Let μ, β and $\{\mu_i$: i$\in \Gamma\}$ be fuzzy sets of a fuzzy topological space $(X, \ \tau)$, the following properties hold:

(i) $\Lambda(1) = 1, V\ (1) = 1$
(ii) $\Lambda(0) = 0, V\ (0) = 0$
(iii) $\mu \ \leq \Lambda(\mu)$ and $\mu \ \geq V(\mu) \Rightarrow \mathrm{V}(\mu) \leq \mu \leq \Lambda(\mu)$,
(iv) $\mu \ \leq \ \beta \Rightarrow \Lambda(\mu) \ \leq \ \Lambda(\beta)$ and $V\ (\mu) \ \leq \ V\ (\beta)$,

(v) $\Lambda(\Lambda(\mu)) = \Lambda(\mu)$ and $V(V(\mu)) = V(\mu)$,

(vi) $\Lambda(\cup \mu_i) = \cup (\Lambda(\mu_i))$ and $V(\cup \mu_i) \geq \cup (V(\mu_i))$,

(vii) $\Lambda(\cap \mu_i) \leq \cap (\Lambda(\mu_i))$ and $V(\cap \mu_i) = \cap (V(\mu_i))$,

(viii) $\Lambda(\mu^c) = (V(\mu))^c$ and $V(\mu^c) = (\Lambda(\mu))^c$.

Lemma 2.4. [13]. For a fuzzy set μ of a fuzzy topological space (X, T),then

(i) If μ is a fuzzy open set, then μ is a fuzzy $\Lambda -$ set ,

(ii) If μ is a fuzzy closed set, then μ is a fuzzy $V -$ set.

The converse of this lemma is not true as shown in [14].

Definition 2.5. [9]. Let $(X,)$ be a fuzzy topological space, then the fuzzy subset μ is called fuzzy $R - \Lambda - set$, if $\mu = \Lambda_R(\mu)$, where

$$\Lambda_R(\mu) = \inf\{\upsilon : \mu \leq \upsilon, \upsilon \text{ is a fuzzy regular open set}\}$$

The complement of a fuzzy $R - \Lambda - set$ is called fuzzy $R - V - set$.

3. Fuzzy $pre - \Lambda -$ and fuzzy $semi - \Lambda - sets$.

Definition 3.1. A fuzzy set λ of a fuzzy topological space X is called:

(i) Fuzzy $pre - \Lambda - set$ $(p - \Lambda - set)$ if $\lambda \geq \lambda^{V\Lambda}$.

(ii) Fuzzy $semi - \Lambda - set$ $(s - \Lambda - set)$ if $\lambda \geq \lambda^{\Lambda V}$

The family of all fuzzy $pre - \Lambda$ (resp. $semi - \Lambda$) sets will be denoted by $Fp - \Lambda(X)$ (resp. $Fs - \Lambda(X)$) .

Theorem 3.2. Let λ be a fuzzy set of a fuzzy topological space (X, T), then the followings hold:

(i) Every fuzzy $\Lambda - set$ is a fuzzy $p - \Lambda - set$,

(ii) Every fuzzy $\Lambda - set$ is a fizzy $s - \Lambda - set$.

(iii) Every fuzzy open set is a fuzzy $p - \Lambda - set$.

(iv) Every fuzzy open set is a fuzzy $s - \Lambda - set$.

Proof.

(i) Let λ be a fuzzy $\Lambda - set$, then $\lambda = \lambda^{\Lambda}$. So $\lambda \geq \lambda^{V}$ implies that $\lambda^{\Lambda} \geq (\lambda^{V})^{\Lambda}$. Thus λ is a fuzzy $p - \Lambda - set$.

(ii) Let λ be a fuzzy $\Lambda - set$, then $\lambda = \lambda^{\Lambda} \geq \lambda^{\Lambda V}$. Hence λ is a fuzzy $s - \Lambda - set$.

The proof of (i) and (ii) come from fact every open set is a fuzzy $\Lambda - set$.

Remark 3.3. The following diagram of the implication is true.

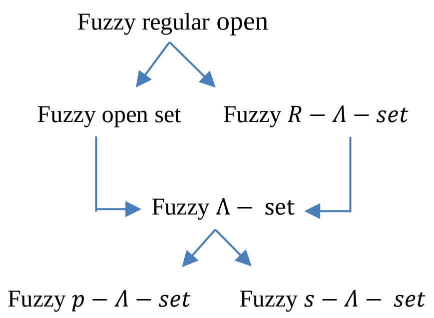

Diagram 1.

The converse of these implications in Diagram1 are not true as shown in the next example.

Example 3.4. Let $X = \{a, b, c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.2$, $\lambda(b) = 0.3$, $\lambda(c) = 0.5$,
$\mu(a) = 0.1$, $\mu(b) = 0.4$, $\mu(c) = 0.5$,
$y(a) = 0.2$, $y(b) = 0.1$, $y(c) = 0.4$.

Consider the fuzzy topology $T = \{0, \lambda, 1\}$. Clearly, μ is a fuzzy $p - \Lambda - set$ because $\mu \geq \mu^{V\Lambda} = 0$. Since $\mu^{\Lambda} \neq \mu$ and hence μ is not a fuzzy $\Lambda - set$. Also y is a fuzzy $s - \Lambda - set$ because $y \geq y^{\Lambda V} = 0$. Since $y^{\Lambda} \neq y$ and hence y is not a fuzzy $\Lambda - set$.

Remark 3.5. The fuzzy $p - \Lambda - sets$ and fuzzy $s - \Lambda - sets$ are independent notions as shown in the following example.

Example 3.6. Let $X = \{a, b, c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = .1$, $\lambda(b) = 0.6$, $\lambda(c) = 0.5$,
$\mu(a) = 0.3$, $\mu(b) = 0.7$, $\mu(c) = 0.3$,
$y(a) = 0$, $y(b) = 0.5$, $y(c) = 0.5$.

Consider the fuzzy topology $T = \{0, \lambda, 1\}$. Clearly, μ is a fuzzy $p - \Lambda - set$ but is not a fuzzy $s - \Lambda - set$ and y is a fuzzy $s - \Lambda - set$ but is not a fuzzy $p - \Lambda - set$.

Theorem 3.7. Let λ be a fuzzy set of a fuzzy topological space (X, T). Then

(i) λ is a fuzzy $p - \Lambda - set$ if and only if there exists a fuzzy $p - \Lambda - set$ υ such that $\lambda^{V} \leq \upsilon \leq \lambda$.

(ii) λ is a fuzzy $s - \Lambda - set$ if and only if there exists a fuzzy $s - \Lambda - set$ υ such that $\upsilon^{V} \leq \lambda \leq \upsilon$.

Proof.

(i) Let λ is a fuzzy $p - \Lambda - set$ of a space (X, T), then $\lambda \geq \lambda^{V\Lambda}$. We put $\upsilon = \lambda^{V\Lambda}$ be a fuzzy $p - \Lambda - set$ and $\lambda \geq \lambda^{V\Lambda} = \upsilon \geq \lambda^{V}$.

Conversely, if v is a fuzzy $p - \Lambda - set$ such that $\lambda^{\vee} \leq v \leq \lambda$, thenn $\lambda \geq v \geq v^{\vee\Lambda} \geq \lambda^{\vee\Lambda}$ and thus λ is a fuzzy $p - \Lambda - set$.

(ii) Let λ is a fuzzy $s - \Lambda - set$ of a space (X, T), then $\lambda \geq \lambda^{\Lambda\vee}$. We put $v = \lambda^{\Lambda}$ be a fuzzy $s - \Lambda - set$ and $v^{\vee} = \lambda^{\Lambda\vee} \leq \lambda \leq \lambda^{\Lambda} = v$.

Conversely, if v is a fuzzy $s - \Lambda - set$ such that $v^{\vee} \leq \lambda \leq v$, then $v \geq v^{\Lambda\vee} \geq \lambda^{\Lambda\vee}$. Since $v^{\vee}$ is the largest fuzzy closed set contained in v, we have $\lambda^{\Lambda\vee} \leq v^{\vee} \leq \lambda$ and hence λ is a fuzzy $s - \Lambda - set$.

Corollary 3.8. Let λ be a fuzzy set of a fuzzy topological space (X, T). Then

(i) λ is a fuzzy $p - \Lambda - set$ if and only if there exists a fuzzy $\Lambda - set$ υ such that $\lambda^{\vee} \leq v \leq \lambda$.

(ii) λ is a fuzzy $s - \Lambda - set$ if and only if there exists a fuzzy $\Lambda - set$ υ such that $v^{\vee} \leq \lambda \leq v$.

Theorem 3.9. Let (X, T) be a fuzzy topological space and $\lambda_i \in F\mathrm{p} - \Lambda(X)$, , then $\cap \{\lambda_i : \mathrm{i} \in I\} \in F\mathrm{p} - \Lambda(X)$, for each $i \in I$.

Proof.

Let λ_i is a fuzzy $p - \Lambda - set$, then $\lambda_i \geq {\lambda_i}^{\vee\Lambda}$ implies that $\cap_{i\in I} \lambda_i \geq \cap_{i\in I} {\lambda_i}^{\vee\Lambda}$

Therefore $\cap_{i\in I} \lambda_i \geq ((\cap_{i\in I} \lambda_i)^{\vee})^{\Lambda} = (\cap_{i\in I} \lambda_i)^{\vee\Lambda}$. Hence $\cap_{i\in I} \lambda_i \in F\mathrm{p} - \Lambda(\mathrm{X})$.

Theorem 3.10. Let (X, T) be a fuzzy topological space and $\lambda_i \in F\mathrm{s} - \Lambda(\mathrm{X})$, then $\cap \{\lambda_i : \mathrm{i} \in I\} \in F\mathrm{s} - \Lambda(\mathrm{X})$, for each $i \in I$.

Proof. Similarly as that of Theorem 3.9.

Remark 3.11. The union of fuzzy $pre - \Lambda -$ (resp. $semi - \Lambda -$) sets need not be a fuzzy $pre - \Lambda -$ (resp. $semi - \Lambda -$) set .This can be shown by the following examples.

Example 3.12. Let $X = \{a, b, c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.6$, $\lambda(b) = 0.4$, $\lambda(c) = 0.8$,

$\mu(a) = 0.5$, $\mu(b) = 0.6$, $\mu(c) = 0.1$,

$y(a) = 0.2$, $y(b) = 0.5$, $y(c) = 0.4$.

Let $T = \{0, \lambda, 1\}$ be a fuzzy topology on X. Clearly μ and y are fuzzy $p - \Lambda - sets$, because $\mu \geq \mu^{\vee\Lambda} = 0$ and $y \geq y^{\vee\Lambda} = 0$. Also we have $(\mu \cup y)^{\vee} = \lambda^{c}$ and $(\mu \cup y) \ngeq (\mu \cup y)^{\vee\Lambda} = 1$. Which shows that $\mu \cup y$ is not a fuzzy $p - \Lambda - set$.

Example 3.13. Let $X = \{a, b, c\}$ and μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of X defined as follows:

$\mu_1(a) = 1$, $\mu_1(b) = 1$, $\mu_1(c) = 0.6$,

$\mu_2(a) = 0.1$, $\mu_2(b) = 0.4$, $\mu_2(c) = 1$,

$\mu_3(a) = 1$, $\mu_3(b) = 0.7$, $\mu_3(c) = 0.6$,

$\mu_4(a) = 0.1$, $\mu_4(b) = 0.3$, $\mu_4(c) = 0.8$.

.Let $T = \{0, \mu_1, \mu_2, \mu_1 \cap \mu_2, 1\}$ be a fuzzy topology on X. Clearly μ_3 and μ_4 are fuzzy $s - \Lambda - sets$, because $\mu_3 \geq {\mu_3}^{\Lambda\vee} = (\mu_1 \cap \mu_2)^{c}$ and $\mu_4 \geq z^{\Lambda\vee} = {\mu_1}^{c}$. Also we have $(\mu_3 \cup \mu_4) \ngeq (\mu_3 \cup \mu_4)^{\Lambda\vee} = 1$. Which shows that $\mu_3 \cup \mu_4$ is not a fuzzy $s - \Lambda - set$.

Lemma 3.14. For every fuzzy set $\lambda \in Fp - \Lambda(X)$, then $\lambda^{\vee} = \lambda^{\vee\Lambda\vee}$.

Proof. Let $\in Fp - \Lambda(X)$, then $\lambda \geq \lambda^{V\Lambda}$ and $\lambda^{\vee} \geq \lambda^{\vee\Lambda\vee}$ thus $\lambda^{\vee} = \lambda^{\vee\Lambda\vee}$.

The next remark gives us the relationship between fuzzy preopen (resp. semi − open) sets with fuzzy pre − Λ − (resp. semi − Λ−) sets.

Remark 3.15. The fuzzy pre − open (resp. semi − open) sets andfuzzy$pre - \Lambda -$ (resp. $semi - \Lambda -$) sets are independent notions. We can show that from the following examples .

Example 3.16. Let $X = \{a, b, c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.6$, $\lambda(b) = 0.4$, $\lambda(c) = 0.8$,

$\mu(a) = 0.3$, $\mu(b) = 0.6$, $\mu(c) = 0.1$,

$y(a) = 0.5$, $y(b) = 0.6$, $y(c) = 0.4$.

Let $T = \{0, \lambda, 1\}$ be a fuzzy topology on X. Then μ is a fuzzy $p - \Lambda - set$ but is not a fuzzy preopen set and y is a fuzzy proopen set but is not a fuzzy $p - \Lambda - set$.

Example 3.17. Let $X = \{a, b, c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.4$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$,

$\mu(a) = 0.2$, $\mu(b) = 0.4$, $\mu(c) = 0.5$,

$y(a) = 0.4$, $y(b) = 0.5$, $y(c) = 0.5$.

Let $T = \{0, \lambda, 1\}$ be a fuzzy topology on X. Then μ is a fuzzy $s - \Lambda - set$ but is not a fuzzy semiopen set and y is a fuzzy semiopen set but is not a fuzzy $s - \Lambda - set$.

4. Fuzzy $pre - V -$ and fuzzy $semi - V - sets$.

Definition 4.1. A fuzzy set λ of a fuzzy topological space is called:

(i) Fuzzy $pre - V - set (p - V - set)$ If $\lambda \leq \lambda^{\Lambda\vee}$.

(ii) Fuzzy $semi - V - set$ ($s - V - set$) if $\lambda \leq \lambda^{\vee\Lambda}$.

The family of all fuzzy $pre - V$ (resp. $semi - V$) sets will be denoted by $Fp - V(X)$ (resp. $Fs - V(X)$) .

Proposition 4.2. Let λ be a fuzzy set of a fuzzy topological space (X,T), then the followings hold:

(i) Every fuzzy $V-set$ is a fuzzy $p-V-set$.

(ii) Every fuzzy $V-\ set$ is a fizzy $s-V-\ set$.

(iii) Every fuzzy closed set is a fuzzy $p-V-set$.

(iv) Every fuzzy closed set is a fuzzy $s-V-set$.

Remark 4.3. The following diagram of the implication is true.

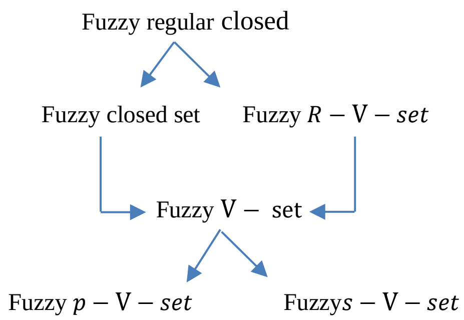

Diagram 2 .

The converse of these implications in Diagram 2 are not true as shown in the next example.

Example 4.4. Let $X\ =\{a,b,c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a)\ =\ 0.2\,,\qquad \lambda(b)\ =\ 0.3,\qquad \lambda(c)\ =\ 0.5,$

$\mu(a)\ =\ 0.9,\qquad \mu(b)\ =\ 0.6,\qquad \mu(c)\ =\ 0.5,$

$y(a)\ =\ 0.8,\qquad y(b)\ =\ 0.9,\qquad y(c)\ =\ 0.6.$

Consider the fuzzy topology $T=\{0,\lambda,1\}$. Clearly, μ is a fuzzy $p-\mathrm{V}-set$ because $\mu\leq\mu^{\Lambda V}=1$. Since $\mu^{V}\neq\mu$ and hence μ is not a fuzzy $V-\ set$. Also y is a fuzzy $s-V-\ set$ because $y\leq y^{V\Lambda}=1$. Since $y^{V}\neq y$ and hence y is not a fuzzy V – set .

Remark 4.5. The fuzzy $p-\mathrm{V}-sets$ and fuzzy $s-\mathrm{V}-sets$ are independent notions as shown in the following example.

Example 4.6. Let $X\ =\{a,b,c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a)\ =\ 1,\qquad \lambda(b)\ =\ 0.6,\qquad \lambda(c)\ =\ 0.5,$

$\mu(a)\ =\ 0.7,\qquad \mu(b)\ =\ 0.3,\qquad \mu(c)\ =\ 0.7,$

$y(a)\ =\ 1,\qquad y(b)\ =\ 0.5,\qquad y(c)\ =\ 0.5.$

Consider the fuzzy topology $T=\{0,\lambda,1\}$. Clearly, μ is a fuzzy $p-\mathrm{V}-set$ but is not a fuzzy $s-\mathrm{V}-\ set$ and y is a fuzzy $s-\mathrm{V}-\ set$ but is not a fuzzy $p-\mathrm{V}-set$.

Theorem 4.7. Let λ be a fuzzy set of a fuzzy space X. Then λ is a fuzzy $p-\mathrm{V}-$ (resp. $s-V-$) set if and only if λ^{c} is a fuzzy $p-\Lambda-$ (resp. $s-\Lambda-$) set.

Proof. Obvious .

Theorem 4.8. Let λ be a fuzzy set of a fuzzy topological space (X,T). Then

(i) λ is a fuzzy $p-\mathrm{V}-set$ if and only if there exists a fuzzy $p-V-set$ υ such that $\lambda\leq v\leq\lambda^{\Lambda}$.

(ii) λ is a fuzzy $s-\mathrm{V}-set$ if and only if there exists a fuzzy s−V−set υ such that $v\leq\lambda\leq v^{\Lambda}$.

Proof.

(i) Let λ is a fuzzy $p-\mathrm{V}-set$ of a space (X,T), then $\lambda\ \leq\lambda^{\Lambda V}$. We put $v\ =\lambda^{\Lambda V}$ be a fuzzy $p-V-set$ and $\lambda\ \leq\ \lambda^{\Lambda V}\ =v\ \leq\lambda^{\Lambda}$.

Conversely, if υ is a fuzzy $p-V-set$ such that $\lambda\leq v\ \leq\lambda^{\Lambda}$,then $\lambda\leq v\leq v^{\Lambda V}\leq\lambda^{\Lambda V}$ and thus λ is a fuzzy $p-\mathrm{V}-set$.

(ii) Let λ is a fuzzy $s-\mathrm{V}-set$ of a space (X,T), then $\lambda\ \leq\lambda^{V\Lambda}$. We put $v\ =\lambda^{V}$ be a fuzzy $s-\mathrm{V}-set$ and $\lambda^{V}=v\leq\lambda\ \leq\lambda^{V\Lambda}=v^{\Lambda}$.

Conversely, if υ is a fuzzy $s-\mathrm{V}-set$ such that $v\leq\lambda\ \leq v^{\Lambda}$, then $\upsilon\ \leq\ v^{V\Lambda}\ \leq\lambda^{V\Lambda}$. Since v^{Λ} is the smallest fuzzy open set contained υ, thus $\lambda\leq v^{\Lambda}\ \leq\lambda^{V\Lambda}$ and hence λ is a fuzzy $s-\mathrm{V}-set$.

Corollary 4.9. Let λ be a fuzzy set of a fuzzy topological space (X,T). Then

(i) λ is a fuzzy $p-\mathrm{V}-set$ if and only if there exists a fuzzy $V\ -set$ υ such that $\lambda\leq v\leq\lambda^{\Lambda}$.

(ii) λ is a fuzzy $s-\mathrm{V}-set$ if and only if there exists a fuzzy $V\ -set$ υ such that $v\ \leq\lambda\leq v^{\Lambda}$.

Theorem 4.10. Let (X,T) be a fuzzy topological space and $\lambda_{i}\ \in F\mathrm{p}-\mathrm{V}(X)$, then $\cup\{\lambda_{i}:\mathrm{i}\ \in I\}\ \in F\mathrm{p}-\mathrm{V}(\mathrm{X})$, for each $i\ \in I$.

Proof.

Let λ_{i} is a fuzzy $p-\mathrm{V}-set$, then $\lambda_{i}\leq{\lambda_{i}}^{\Lambda V}$ implies that $\cup_{i\in I}\lambda_{i}\leq\cup_{i\in I}{\lambda_{i}}^{\Lambda V}$ therefore$\cup_{i\in I}\ \lambda_{i}\leq((\cup_{i\in I}\ \lambda_{i})^{\Lambda})^{V}=(\cup_{\mathrm{i}\in I}\lambda_{i})^{\Lambda v}$. Hence $\cup_{i\in I}\lambda_{i}\ \in F\mathrm{p}-\mathrm{V}(\mathrm{X})$.

Theorem 4.11. Let (X,T) be a fuzzy topological space and $\lambda_{i}\in Fs-\mathrm{V}(\mathrm{X})$, then $\cup\{\lambda_{i}:\mathrm{i}\ \in I\}\ \in Fs-\mathrm{V}(\mathrm{X})$, for each $i\ \in I$.

Proof. Similarly as that of proof 4.10.

Remark 4.12. The intersection of fuzzy $pre-\mathrm{V}-$ (resp. $semi-\mathrm{V}-$) sets need not be a fuzzy $pre-\mathrm{V}-$ (resp. $semi-\mathrm{V}-$) set .This can be shown by the following examples.

Example 4.13. Let λ, μ and y be fuzzy sets of fuzzy space (X,T) as defined in Example 3.12. From Example 3.12, and Theorem 4.7, we get μ^{c} and y^{c} are fuzzy $p-\mathrm{V}-sets$. But $\mu^{c}\cap y^{c}=(\mu\ \cup\ \mathrm{y})^{c}$ is not a fuzzy $p-\mathrm{V}-set$, because $\mu\cup y$ is not a fuzzy $p-\Lambda-set$.

Example 4.14. Let μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of fuzzy space (X,T) as defined in Example 3.13. From Example 3.13, and Theorem 4.7, we get $\mu_3{}^c$ and $\mu_4{}^c$ are fuzzy $s-V-sets$. But $\mu_3{}^c \cap \mu_4{}^c = (\mu_3 \cup \mu_4)^c$ is not a fuzzy $s-V-set$, because $\mu_3 \cup \mu_4$ is not a fuzzy $s-\Lambda-set$.

Theorem 4.15. Let λ be a fuzzy set of a fuzzy space X. Then λ is a fuzzy $s-V-set$ if and only if $\lambda^{\Lambda} = \lambda^{V\Lambda}$.

Proof.

Let λ be a fuzzy $s-V-set$, then $\lambda \le \lambda^{V\Lambda}$ implies $\lambda^{\Lambda} \le (\lambda^{V\Lambda})^{\Lambda} = \lambda^{V\Lambda}$. Therefore, $\lambda^{\Lambda} = \lambda^{V\Lambda}$. Conversely, let λ be a fuzzy set such that $\lambda^{\Lambda} = \lambda^{V\Lambda}$, then $\lambda \le \lambda^{\Lambda} = \lambda^{V\Lambda}$, that is λ is a fuzzy $s-V-set$.

The next remark gives us the relationship between fuzzy preclosed $(\text{resp. semi}-\text{closed})$ sets with fuzzy $\text{pre}-V-$ $(\text{resp. semi}-V-)$ sets.

Remark 4.16. The fuzzy $\text{pre}-\text{closed}$ $(\text{resp. semi}-\text{closed})$ sets and fuzzy $pre-V-$ $(\text{resp.} \, semi-V-)$ sets are independent notions. We can show that from the following examples.

Example 4.17. Let $X = \{a,b,c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.6$, $\lambda(b) = 0.4$, $\lambda(c) = 0.8$,

$\mu(a) = 0.7$, $\mu(b) = 0.4$, $\mu(c) = 0.9$,

$y(a) = 0.5$, $y(b) = 0.4$, $y(c) = 0.6$.

Let $T = \{0, \lambda, 1\}$ be a fuzzy topology on X. Then μ is a fuzzy $p-V-set$ but is not a fuzzy preclosed set and y is a fuzzy preclosed set but is not a fuzzy $p-V-set$.

Example 4.18. Let $X = \{a,b,c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.4$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$,

$\mu(a) = 0.8$, $\mu(b) = 0.6$, $\mu(c) = 0.5$,

$y(a) = 0.6$, $y(b) = 0.5$, $y(c) = 0.5$.

Let $T = \{0, \lambda, 1\}$ be a fuzzy topology on X. Then μ is a fuzzy $s-V-set$ but is not a fuzzy semiclosed set and y is a fuzzy semiclosed set but is not a fuzzy $s-V-set$.

5. Some operators using fuzzy $\text{pre}-\Lambda-$ $(\text{resp. fuzzy pre}-V-)$ sets and fuzzy $\text{semi}-\Lambda-$ $(\text{resp. fuzzy semi}-V-)$ sets.

Definition 5.1. Let λ be a fuzzy set of a fuzzy topological space X. Then the fuzzy sets $p_{\Lambda}(\lambda)$ and $s_{\Lambda}(\lambda)$ are defined as:

$p_{\Lambda}(\lambda) = \cap \{v : v \ge \lambda, v \text{ is a fuzzy } pre-\Lambda-set\}$

And $s_{\Lambda}(\lambda) = \cap \{v : v \ge \lambda, v \text{ is a fuzzy } semi-\Lambda-set\}$.

We can say that $p_{\Lambda}(\lambda)$ and $s_{\Lambda}(\lambda)$ is the smallest fuzzy $pre-\Lambda-set$ and $semi-\Lambda-set$ of X containing λ, respectively.

Theorem 5.2. Let λ be a fuzzy set of a fuzzy space X. Then

(i) λ is a fuzzy $pre-\Lambda-set$ if and only if $\lambda = p_{\Lambda}(\lambda)$,

(ii) λ is a fuzzy $semi-\Lambda-set$ if and only if $\lambda = s_{\Lambda}(\lambda)$,

proof.

(i) Let λ be a fuzzy $p-\Lambda-set$ of X, then λ is the smallest fuzzy $p-\Lambda-set$ contains itself. Thus $\lambda = p_{\Lambda}(\lambda)$. Conversely, if $\lambda = p_{\Lambda}(\lambda)$, then obviously λ is a fuzzy $pre-\Lambda-set$.

(ii) The proof is similar to the proof of (i).

Theorem 5.3. Let λ, μ and λ_i $(i \in I)$ be fuzzy sets of a fuzzy space X. Then the followings hold:

(i) $\lambda \le p_{\Lambda}(\lambda) \le \lambda^{\Lambda}$, $\lambda \le s_{\Lambda}(\lambda) \le \lambda^{\Lambda}$;

(ii) If $\lambda \le \mu$, then $p_{\Lambda}(\lambda) \le p_{\Lambda}(\mu)$ and $s_{\Lambda}(\lambda) \le s_{\Lambda}(\mu)$;

(iii) $p_{\Lambda}(p_{\Lambda}(\lambda)) = p_{\Lambda}(\lambda)$ and $s_{\Lambda}(s_{\Lambda}(\lambda)) = s_{\Lambda}(\lambda)$;

(iv) $p_{\Lambda}\{\cap\{\lambda_i, i \in I\}) \le \cap\{p_{\Lambda}(\lambda_i), i \in I\}$ and $s_{\Lambda}\{\cap\{\lambda_i, i \in I\}) \le \cap\{s_{\Lambda}(\lambda_i), i \in I\}$;

(v) $p_{\Lambda}\{\cup\{\lambda_i, i \in I\}) \ge \cup\{p_{\Lambda}(\lambda_i), i \in I\}$ and $s_{\Lambda}\{\cup\{\lambda_i, i \in I\}) \ge \cup\{s_{\Lambda}(\lambda_i), i \in I\}$.

Proof.

We shall only consider the case of $p_{\Lambda}(\lambda)$.

(i) Since $\lambda \le \lambda^{\Lambda}$, then $\lambda \le p_{\Lambda}(\lambda) \le p_{\Lambda}(\lambda^{\Lambda})$ and by Theorem 5.2, $p_{\Lambda}(\lambda^{\Lambda}) = \lambda^{\Lambda}$ that is $\lambda \le p_{\Lambda}(\lambda) \le \lambda^{\Lambda}$.

(ii) If $\lambda \le \mu$ and by (i) we get $\lambda \le \mu \le p_{\Lambda}(\mu)$. Since $\lambda \le p_{\Lambda}(\lambda)$ and $p_{\Lambda}(\lambda)$ is the smallest fuzzy $p-\Lambda-set$ contains λ. Hence $p_{\Lambda}(\lambda) \le p_{\Lambda}(\mu)$.

(iii) Let $\mu = p_{\Lambda}(\lambda)$. Since $p_{\Lambda}(\mu)$ is a fuzzy $p-\Lambda-set$, then $p_{\Lambda}(\mu) = \mu = p_{\Lambda}(\lambda)$. Thus $p_{\Lambda}(p_{\Lambda}(\lambda) = p_{\Lambda}(\lambda)$.

(iv) Let $\lambda = \cap\{\lambda_i, i \in I\}$. Then $\lambda \le \lambda_i$, for each i∈ I, then from (ii) we have $p_{\Lambda}(\lambda) \le p_{\Lambda}(\lambda_i)$, for each $i \in I$, therefore $p_{\Lambda}(\cap\{\lambda_i, i \in I\}) \le \cap \{p_{\Lambda}(\lambda_i), \text{i} \in I\}$.

(v) Let $\lambda = \cup\{\lambda_i, i \in I\}$. Then $\lambda \ge \lambda_i$, for each i∈ I, then from (ii) we have $p_{\Lambda}(\lambda) \ge p_{\Lambda}(\lambda_i)$, for each i∈ I, therefore $p_{\Lambda}(\cup\{\lambda_i, i \in I\}) \ge \cup \{p_{\Lambda}(\lambda_i), \text{i} \in I\}$.

Remark 5.4. In Theorem 5.3, the inclusion (iv) and (v) cannot be replaced by equality. It can be shown in following examples.

Example 5.5. Let $X = \{a, b, c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.2$, $\lambda(b) = 0.6$, $\lambda(c) = 0.8$,

$\mu(a) = 0.1$, $\mu(b) = 1$, $\mu(c) = 0.6$,

$y(a) = 0.8$, $y(b) = 0.6$, $y(c) = 0.7$.

Let $T = 0, \lambda, 1$ be a fuzzy topology on X. Then can be say that $p_\Lambda(\mu \cap y) = (\mu \cap y) \neq \mu = p_\Lambda(\mu) \cap p_\Lambda(y)$ and $s_\Lambda(\mu \cap y) = (\mu \cap y) \neq 1 = s_\Lambda(\mu) \cap s_\Lambda(y)$.

Example 5.6. Let λ, μ and y be fuzzy sets of fuzzy space (X, T) as defined in Example 3.12, from Example 3.12, and Theorem 5.2, we get $p_\Lambda(\mu) = \mu$ and $p_\Lambda(y) = y$ but $p_\Lambda(\mu \cup y) = 1 \neq \mu \cup y = p_\Lambda(\mu) \cup p_\Lambda(y)$.

Example 5.7. Let μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of fuzzy space (X, T) as defined in Example 3.13, from Example 3.13, and Theorem 5.2, we get $s_\Lambda(\mu_3) = \mu_3$ and $s_\Lambda(\mu_4) = \mu_4$ but $s_\Lambda(\mu_3 \cup \mu_4) = 1 \neq \mu_3 \cup \mu_4 = s_\Lambda(\mu_3) \cup s_\Lambda(\mu_4)$.

Definition 5.8. Let λ be a fuzzy set of a fuzzy topological space X. Then the fuzzy sets $p_V(\lambda)$ and $s_V(\lambda)$ are defined as:

$p_V(\lambda = \cup \{v : v \leq \lambda, v \text{ is a fuzzy } pre - V - set\}$

And $s_V(\lambda) = \cup [v : v \leq \lambda, v \text{ is a fuzzy } semi - V - set\}$.

We can say that $p_V(\lambda)$ and $s_V(\lambda)$ is the largest fuzzy $pre - V - set$ and $semi - V - set$ of X contained in λ, respectively.

Theorem 5.9. Let λ be a fuzzy set of a fuzzy space X. Then

(i) λ is a fuzzy $pre - V - set$ if and only if $\lambda = p_V(\lambda)$,
(ii) λ is a fuzzy $semi - V - set$ if and only if $\lambda = s_V(\lambda)$.

Proof.

(i) Let λ be a fuzzy $p - V - set$ of X, then λ is the largest fuzzy $p - V - set$ contained in itself. Thus $\lambda = p_V(\lambda)$. Conversely, if $\lambda = p_V(\lambda)$, then obviously λ is a fuzzy $pre - V - set$.
(ii) The proof is similar to the proof of (i).

Theorem 5.10. Let λ, μ and λ_i $(i \in I)$ be fuzzy sets of a fuzzy space X. Then the followings hold:

(i) $\lambda^V \leq p_V(\lambda) \leq \lambda$, $\lambda^V \leq s_V(\lambda) \leq \lambda$;
(ii) If $\lambda \leq \mu$, then $p_V(\lambda) \leq p_V(\mu)$ and $s_V(\lambda) \leq s_V(\mu)$;
(iii) $p_V(p_V(\lambda)) = p_V(\lambda)$ and $s_V(s_V(\lambda)) = s_V(\lambda)$;
(iv) $p_V(\cap \{\lambda_i, i \in I\}) \leq \cap \{p_V(\lambda_i), i \in I\}$ and $s_V(\cap \{\lambda_i, i \in I\}) \leq \cap \{s_V(\lambda_i), i \in I\}$;
(v) $p_V(\cup \{\lambda_i, i \in I\}) \geq \cup \{p_V(\lambda_i), i \in I\}$ and $s_V(\cup \{\lambda_i, i \in I\}) \geq \cup \{s_V(\lambda_i), i \in I\}$.

Proof.

We shall only consider the case of $p_V(\lambda)$.

(i) Since $\lambda^V \leq \lambda$, then $p_V(\lambda^V) \leq p_V(\lambda) \leq \lambda$ and by Theorem 5.9, $p_V(\lambda^V) = \lambda^V$,that is $\lambda^V p_V(\lambda) \leq \lambda$
(ii) If $\lambda \leq \mu$ and by (i) we have $p_V(\lambda) \leq \lambda \leq \mu$. Since $p_V(\mu) \leq \mu$ and $p_V(\mu)$ is the largest fuzzy $p - V - set$ contained in μ. Thus $p_V(\lambda) \leq p_V(\mu)$.
(iii) Let $\mu = p_V(\lambda)$. Since $p_V(\mu)$ is a fuzzy $p - V -$ set, then $p_V(\mu) = \mu = p_V(\lambda)$. Thus $p_V(p_V(\lambda)) = p_V(\lambda)$.
(iv) Let $\lambda = \cap \{\lambda_i, i \in I\}$. Then $\lambda \leq \lambda_i$, for each $i \in I$, then from (ii) we have $p_V(\lambda) \leq p_V(\lambda_i)$, for each i$\in I$, therefore $p_V(\cap \{\lambda_i, i \in I\}) \leq \cap \{p_V(\lambda_i), i \in I\}$.
(v) Let $\lambda = \cup \{\lambda_i, i \in I\}$. Then $\lambda \geq \lambda_i$, for each $i \in I$, then from (ii) we have $p_V(\lambda) \geq p_V(\lambda_i)$, for each $i \in I$, therefore $p_V(\cup \{\lambda_i, i \in I\} \geq \cup \{p_V(\lambda_i), i \in I\}$.

Remark 5.11. In Theorem 5.10, the inclusion (iv) and (v) cannot be replaced by equality. It can be shown in following examples.

Example 5.12. Let $X = \{a, b, c\}$ and λ, μ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.2$, $\lambda(b) = 0.6$, $\lambda(c) = 0.8$,
$\mu(a) = 0.9$, $\mu(b) = 0$, $\mu(c) = 0.4$,
$y(a) = 0.2$, $y(b) = 0.4$, $y(c) = 0.3$.

Let $\tau = \{0, \lambda, 1\}$ be a fuzzy topology on X. Then can be say that $p_V(\mu \cup y) = (\mu \cup y) \neq \mu = p_V(\mu) \cup p_V(y)$ and $s_V(\mu \cup y) = (\mu \cup y) \neq 0 = s_V(\mu) \cup s_V(y)$.

Example 5.13. Let λ, μ and y be fuzzy sets of fuzzy space (X, τ) as defined in Example 5.13, from Example 5.13, and Theorem 5.9, we get $p_V(\mu^c) = \mu^c$ and $p_V(y^c) = y^c$ but $p_V(\mu^c \cap y^c) = 0 \neq \mu^c \cap y^c = p_V(\mu^c) \cap p_V(y^c)$.

Example 5.14. Let μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of fuzzy space (X, τ) as defined in Example 5.14, from Example 5.14. and Theorem 5.9, we get $s_V({\mu_3}^c) = {\mu_3}^c$ and $s_V({\mu_4}^c) = {\mu_4}^c$but $s_V({\mu_3}^c \cap {\mu_4}^c) = 0 \neq {\mu_3}^c \cap {\mu_4}^c = s_V({\mu_3}^c) \cap s_V({\mu_4}^c)$.

Theorem 5.15. Let λ be a fuzzy set of a fuzzy topological space X. Then

(i) $(p_\Lambda(\lambda))^c = p_V(\lambda^c)$, $(s_\Lambda(\lambda))^c = s_V(\lambda^c)$.
(ii) $(p_V(\lambda))^c = p_V(\lambda^c)$, $(s_V(\lambda))^c = s_\Lambda(\lambda^c)$.

Proof.

We shall prove only $(p_\Lambda(\lambda))^c = p_V(\lambda^c)$. We have

$(p_\Lambda(\lambda))^c = (\cap \{\upsilon : \upsilon \ \ \upsilon \text{ is a fuzzy } pre - \Lambda - \text{set}\})^c$

$= \cup \{\upsilon^c : \upsilon^c \leq \lambda^c, \upsilon^c \text{ is a fuzzy } pre - V - \text{set}\}$

$= p_V(\lambda^c)$.

Theorem 5.16. Let λ be a fuzzy set of a fuzzy topological space X. Then

(i) $p_\Lambda(\lambda) \geq \lambda \cup \lambda^{\vee\Lambda}$,
$p_\vee(\lambda) \leq \lambda \cap \lambda^{\Lambda\vee}$
(ii) $s_\Lambda(\lambda) = \lambda \cup \lambda^{\Lambda\vee}$.
$s_\vee(\lambda) = \lambda \cap \lambda^{\Lambda\vee}$.

Proof.

(i) Since $\lambda \leq p_\Lambda(\lambda)$ and $p_\Lambda(\lambda)$ is a fuzzy $p - \Lambda - set$. Then $p_\Lambda(\lambda) \geq (p_\Lambda(\lambda))^{\vee\Lambda} \geq \lambda^{\vee\Lambda}$, thus $p_\Lambda(\lambda) \geq \lambda^{\vee\Lambda}$ and $p_\Lambda(\lambda) \geq \lambda \cup \lambda^{\vee\Lambda}$. Also since $\lambda \geq p_\vee(\lambda)$ and $p_\vee(\lambda)$ is a fuzzy $p - V - set$. Then $p_\vee(\lambda) \leq (p_\vee(\lambda))^{\Lambda\vee} \leq \lambda^{\Lambda\vee}$, thus $p_\vee(\lambda) \leq \lambda^{\Lambda\vee}$ and $p_\vee(\lambda) \leq \lambda \cap \lambda^{\Lambda\vee}$.

(ii) Since $\lambda \leq s_\Lambda(\lambda)$ and $s_\Lambda(\lambda)$ is a fuzzy $s - \Lambda - set$. Then $s_\Lambda(\lambda) \geq (s_\Lambda(\lambda))^{\Lambda\vee} \geq \lambda^{\Lambda\vee}$, thus $s_\Lambda(\lambda) \geq \lambda^{\Lambda\vee}$ and $s_\Lambda(\lambda) \geq \lambda \cup \lambda^{\Lambda\vee}$. Since $\lambda^{\Lambda\vee} \leq \lambda \cup \lambda^{\Lambda\vee} \leq \lambda^\Lambda$ and by Corollary 3.8, we get $\lambda \cup \lambda^{\Lambda\vee}$ is a fuzzy $s - \Lambda - set$ and $\lambda \cup \lambda^{\Lambda\vee} \geq \lambda$. Thus $\lambda \cup \lambda^{\Lambda\vee} \geq s_\Lambda(\lambda)$ and hence $s_\Lambda(\lambda) = \lambda \cup \lambda^{\Lambda\vee}$.

The proof of other part is similar.

Remark 5.17. In Theorem 5.16, the inclusion (i) cannot be replaced by equality. It can be shown in next example.

Example 5.18. Let $X = \{a, b, c\}$ and λ, µ and y be fuzzy sets of X defined as follows:

$\lambda(a) = 0.7$, $\lambda(b) = 0.8$, $\lambda(c) = 0.2$,

$\mu(a) = 0.2$, $\mu(b) = 0.2$, $\mu(c) = 0.6$,

$y(a) = 0.8$, $y(b) = 0.7$, $y(c) = 0.6$.

Let $\tau = \{0, \lambda, \mu, \lambda \cap \mu, \lambda \cup \lambda, 1\}$ be a fuzzy topology on X. Then can be say that $p_\Lambda(y) = 1$ and $y \cup y^{\vee\Lambda}$ is not equal to 1 . And by Lemma 2.3, (vii) and Theorem 5.15, we get $p_\vee(y^c) \neq y^c \cap (y^c)^{\Lambda\vee}$.

Theorem 5.19. Let λ be a fuzzy set of a fuzzy topological space X. Then

the following statements hold.

(i) $p_\Lambda(\lambda^\vee) = \lambda^{\vee\Lambda}$, $s_\Lambda(\lambda^\vee) = \lambda^{\vee\Lambda}$
(ii) $p_\vee(\lambda^\Lambda) = \lambda^{\Lambda\vee}$, $s_\vee(\lambda^\Lambda) = \lambda^{\Lambda\vee}$
(iii) $p_\Lambda(\lambda))^\Lambda = p_\Lambda(\lambda^\Lambda) = \lambda^\Lambda$,
$(s_\Lambda(\lambda))^\Lambda = s_\Lambda(\lambda^\Lambda) = \lambda^\Lambda$
(iv) $(p_\vee(\lambda))^\vee = p_\vee(\lambda^\vee) = \lambda^\vee$
$(s_\vee(\lambda))^\vee = s_\Lambda(\lambda^\vee) = \lambda^\vee$

Proof.

We shall only consider the case of $p_\Lambda(\lambda)$ and $p_\vee(\lambda)$.

(i) By Theorem 5.3, we have $p_\Lambda(\lambda^\vee) \leq \lambda^{\vee\Lambda}$. …… (1)

Also we have $p_\Lambda(\lambda^\vee)$ is a fuzzy $p - \Lambda - set$, that is
$p_\Lambda(\lambda^\vee) \geq (p_\Lambda(\lambda^\vee))^{\vee\Lambda}$
$\geq (\lambda^\vee)^{\vee\Lambda}$
$= \lambda^{\vee\Lambda}$. …….. (2)

From (1) and (2) we get $p_\Lambda(\lambda^\vee) = \lambda^{\vee\Lambda}$.

(ii) By Theorem 5.10, we have $p_\vee(\lambda^\Lambda) \geq \lambda^{\Lambda\vee}$. ……. (3)

Also we have $p_\vee(\lambda^\Lambda)$ is a fuzzy $p - V - set$, that is
$p_\vee(\lambda^\Lambda) \leq (p_\Lambda(\lambda^\Lambda))^{\Lambda\vee}$
$\leq (\lambda^\Lambda)^{\Lambda\vee}$
$= \lambda^{\Lambda\vee}$. ………(4)

From (3) and (4) we get $p_\vee(\lambda^\Lambda) = \lambda^{\Lambda\vee}$.

(iii) Since $\lambda \leq p_\Lambda(\lambda)$, so $\lambda^\Lambda \leq (p_\Lambda(\lambda))^\Lambda$ and we have $p_\Lambda(\lambda) \leq \lambda^\Lambda$, so $(p_\Lambda(\lambda))^\Lambda \leq \lambda^\Lambda$, thus $(p_\Lambda(\lambda))^\Lambda = \lambda^\Lambda$. Also by Theorem 5.2, we get $p_\Lambda(\lambda^\Lambda) = \lambda^\Lambda$. Hence $(p_\Lambda(\lambda))^\Lambda = p_\Lambda(\lambda^\Lambda) = \lambda^\Lambda$.

(iv) Since $\lambda \geq p_\vee(\lambda)$, so $\lambda^\vee \geq (p_\vee(\lambda))^\vee$ and we have $p_\vee(\lambda) \geq \lambda^\vee$, so $(p_\vee(\lambda))^\vee \geq \lambda^\vee$, thus $(p_\vee(\lambda))^\vee = \lambda^\vee$. Also by Theorem 5.9, we get $p_\vee(\lambda^\vee) = \lambda^\vee$. Hence $(p_\vee(\lambda))^\vee = p_\vee(\lambda^\vee) = \lambda^\vee$.

References

[1] L. Zadeh, "Fuzzy sets," Information and Control, (8) (3), 338–353, 1965.

[2] C. L. Chang, Fuzzy topological spaces, Journal of Mathematical Analysis and Applications, 245, 182-190, 1968.

[3] W. Shi. & K. A, Liu, fuzzy topology for computing the interior, boundary, and exterior of spatial objects quantitatively in GIS. Comput Geosci. 33:898–915, 2007.

[4] N. Aimal and B. K. Tyagi, On fuzzy almost continuous functions, Fuzzy Sets and Systems 41, 221-232, 1991.

[5] R. Badard, Fuzzy pretopological spaces and their representation, Journal of Mathematical Analysis and Applications, 81, 378-390, 1981.

[6] M. Caldas and G. Navalagi, On weak forms of preopen and preclosed functions, Archivum Mathematicum , (BRNO) , 40 , 119-128, 2004 .

[7] Y. C. Kim, A. A. Ramadan and S. E. Abbas, r-fuzzy strongly preopen sets in fuzzy topological spaces Mat.Vesnik 55, 1-13, 2003.

[8] S. S. Thakur and S. Singh, On fuzzy semi-preopen sets and fuzzy semi-precontinuity, Fuzzy Sets and Systems 98, 383-391, 1998.

[9] M. E. El Shafei and A. Zakari, θ-Generalized closed sets in fuzzy topological spaces, The Arabian Journal for Science and Engineering 31, N 24, (2006).

[10] S, B, Halder, On fuzzy R - Λ and fuzzy R – V sets, Studii Si Cercetari Stiintifice Seria Mathematical, Nr 18, 13 – 30, 2008.

[11] K. K. Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, Journal of Mathematical Analysis and Applications, 82, 14-32, 1981.

[12] M. E. Abd El-Monsef and M. H. Ghanim, On semiopen fuzzy sets, Delta J. of Sci., 5 ,30-44, 1981.

[13] A. S. Bin Shahna, On fuzzy strong semicontinuity and fuzzy pre-continuity, Fuzzy Sets and Systems, 44. 303-308, 1991.

[14] K. Balasubramaniyan, S. Sriram, and O. Ravi, “ON Λ-Generalized closed sets in fuzzy topological spaces." Jordan Journal of Mathematics and Statistics *(jjms)* 7.1, 29-46, 2014.

مقالة بحثية

حول المجموعات من النوع $pre-\Lambda-$sets و $semi-\Lambda-$sets الضبابية

رضوان محمد عقيل[1] و أنهار أحمد ناصر[1]،*

[1] *قسم الرياضيات، كلّيّة العلوم عدن، جامعة عدن، عدن، اليمن؛ البريد الالكتروني: raqeel1976@yahoo.com*

*** الباحث الممثّل: أنهار أحمد ناصر؛ البريد الالكتروني: anharnasser724@gmail.com**

استلم في: 16 مارس 2023 / قبل في: 24 مارس 2023 / نشر في 31 مارس 2023

المُلخّص

ناقش هذا البحث أنواع جديدة من المجموعات الشبه مفتوحة بالاعتماد على مفهوم المجموعات من نوع ($\Lambda-$set) و (V$-$set) الضبابية. قمنا بتعريف أنماط جديدة وهي ($pre-\Lambda-$set)، ($pre-$V$-$set)، ($semi-\Lambda-$set) و ($semi-$V$-$set) الضبابية. تمت دراسة الخواص والخصائص للمفاهيم المطروحة في هذا البحث إضافة الى ذلك تمت دراسة العلاقة بين المفاهيم وبعض المفاهيم المعروفة. أخيرا قمنا بتعريف داخلية وخارجية هذه المفاهيم وأنشأنا خواصها المختلفة.

الكلمات المفتاحية: المجموعات من النوع Λ-set، $pre-$Λ$-$set و $semi-\Lambda-$sets، $pre-$V$-$set و $semi-$V$-$sets الضبابية.

How to cite this article:

R. M. Aqeel and A. A. Nasser, “On Fuzzy Pre-Λ-Sets and Fuzzy Semi-Λ- Sets”, *Electron. J. Univ. Aden Basic Appl. Sci.*, vol. 4, no. 1, pp. 139-146, Mar. 2023. DOI: https://doi.org/10.47372/ejua-ba.2023.1.229